The Time-Consistent Optimal Export Policy, Market Structure, and Time-Non-Separable Preferences

Orlando Balboa,[a] Robert Driskill[*],[b] and Andrew Horowitz[c]

[a]USACH University, Chile

[b]Vanderbilt University

[c]University of Arkansas

Abstract

The interplay of time-non-separable (TNS) utility and non-competitive market structure gives rise to a time-consistency issue that changes the optimal export policy prescription vis-à-vis what would obtain in either a time-separable utility setting or in a TNS setting in which firms could credibly commit to a path of output. In particular, in the time-consistent equilibrium both the traditional terms-of-trade exploitation motive for an export tax and the profit-shifting strategic-trade analysis motive for an export subsidy coexist, and the optimal policy prescription depends critically on demand and cost parameters.

JEL Classification: F12

Keywords: Strategic trade, optimal export policy, dynamic demand

1. Introduction

Static analysis of the optimal export policy for promotion of national welfare has generally focused on two motives. One of these is known as a "standard" terms-of-trade motive: if a country's government could restrict output of a non-monopolistic export industry by imposition of an export tax that would raise the world price, then the country's total net profits, i.e., industry profits plus export tax revenue, could increase.[1] This

---

[1] Quotes have been put around "standard" because it is not certain where this argument first made an appearance. It is referred to as the "standard" terms-of-trade argument in Section IV of Eaton and Grossman (1986), in Brander (1995), p. 1411, and in textbooks.

terms-of-trade motive is usually analyzed in policy environments in which there are no strategic considerations vis-à-vis firms located in other countries. In the typical static model used in these analyses, if the demand curve for a country's export good is downward sloping, and if any foreign production of the good is produced under competitive conditions with increasing marginal cost, the optimal domestic policy is an export tax that makes the domestic export industry mimic a monopolist vis-à-vis the rest of the world. This implies, of course, that if the domestic industry is already a monopoly, this tax would be zero.[2]

The second motive, usually referred to as the profit-shifting motive, arises in oligopoly settings in which a government's ability to credibly commit to an export subsidy or tax allows their exporters to move along the foreign firms' reaction functions so as to increase the home country's total net profits. Optimal prescriptions produced from such analyses are referred to as strategic trade policies. In the archetypal static model used in such strategic trade analyses, two Nash-Cournot duopolists, one a home-country firm and one a foreign-country firm, compete in a third-country market. By providing the domestic firm with an export subsidy, the home government shifts net profits to the home country and improves national welfare. The key idea here is that the foreign firm views the subsidy as a credible commitment and thus behaves as if the home-country firm has a cost advantage.

The two motives coexist in expansions of the archetypal strategic trade models in which there is more than one domestic firm. Dixit (1984) first addressed this issue in a Nash-Cournot oligopoly model, and concluded that for a linear model the optimal policy was a subsidy if the number of foreign firms exceed the number of domestic firms by at least 1. Eaton and Grossman (1986, Section IV), and Krishna and Thursby (1991) also analyzed the multi-firm oligopolistic case and concluded that the coexistence of the two motives made possible either a strategic tax or subsidy, depending on the specifics of the model, as long as there was more than one domestic firm. Similarly, Brander (1995, p. 1411) looked at the case of more than one domestic firm and noted that, regardless of the number of foreign firms, these domestic firms would be rivals among themselves as well as rivals with the foreign firms. Thus, a domestic subsidy would increase each of the domestic firms' outputs, and consequently decrease the profit of each. He pointed out that if the number of domestic firms were large relative to the number of foreign firms, national welfare would likely be "enhanced by an export tax, moving domestic firms closer to the cartel output." He further noted that "this is just the standard terms of trade argument for intervention."

All of these analyses are static, although Eaton and Grossman (1986, footnote 2) argued that their use of conjectural variations is an attempt to "collapse the outcome ... of a dynamic process into a static formulation." Dynamic extensions of strategic trade analysis include Driskill and McCafferty (1989), who analyzed the Markov perfect equilibrium (roughly speaking, the time-consistent equilibrium), of a duopoly in which demand is static but firms face adjustment costs in changing their levels of output. They showed that the steady state of the Markov perfect equilibrium of this game corresponded to the

---

[2] The basic idea has been extended to account for non-identical domestic firms (Rodrik, 1989) and general-equilibrium effects (De Santis, 2000).

equilibrium of an analogous static conjectural variations duopoly model in which the conjectural variation was greater than the Nash-Cournot (zero) conjecture and less than the "consistent" conjecture. Thus, in accordance with Eaton and Grossman's finding that the optimal policy in a conjectural variation's duopoly model was a subsidy if the conjectural variation was greater than the Nash-Cournot (zero) conjecture and less than the "consistent" conjecture, they found that the optimal policy in this setting was a subsidy.

In an extension in which dynamics entered the structural model from the demand side, Driskill and Horowitz (1996) analyzed strategic trade in a Markov perfect equilibrium between two duopolists each of whom produced the same durable good under conditions of increasing marginal cost. In this analysis, the steady state of the Markov perfect equilibrium had greater output and lower price than the corresponding steady state of a commitment equilibrium, i.e., an equilibrium in which firms can commit to a path of output for all time. The optimal export policy in this analysis was a tax.

As noted by Reinganum and Stokey (1985), dynamic models in which agents can commit are essentially static. Thus, these dynamic extensions suggest that the steady state Markov perfect equilibrium of two-firm dynamic models can be interpreted as isomorphic to the equilibria of analogous static multi-firm models.[3] This implies, by analogy with the results of the multi-firm static models mentioned previously, that in the equilibrium of the Markov perfect equilibrium of the aforementioned two-firm dynamic models, the two aforementioned motives coexist.

This paper exploits this observation and uses it to understand optimal export policy analyses in an environment in which demand is derived from intertemporally dependent preferences. These type of preferences, introduced by Ryder and Heal (1973), similar to those in Pollak (1970), and with variants used by, among others, Becker and Murphy (1988) and Driskill and McCafferty (2001), specify that utility from current consumption depends on both current and past consumption. This specification leads to intertemporally-linked demand functions that parametrically describe a range of behaviors stretching from demand for durables to demand characterized by Pollak (1970) as arising from "habit formation" and characterized by Becker and Murphy as describing "addictive behavior" and "learning by doing" on the consumer's part. An alternative description of these ranges of behavior would dichotomize them into those that exhibit "adjacent substitutability" in time and those that exhibit "adjacent complementarity" in time. Roughly speaking, demand for durable goods is isomorphic to adjacent substitutability, and "habit formation", (or equivalently, "learning by doing" or "addictive behavior") is isomorphic to adjacent complementarity.[4]

---

[3] Perhaps the most intuitive description of this feature of Markov perfect equilibria comes from Coase (1972), who noted how a time-consistent durable good monopolist acts like a competitive industry. More generally, a backward-induction equilibrium for a monopolist can be interpreted as leader-follower competition between the a last-period monopolist and earlier-period monopolists.

[4] The form of demand from this specification is also isomorphic to what arises from an overlapping-generations framework in which there are either congestion effects or positive network externalities. The results of this paper when there are congestion effects are similar to those in Yuen and Zang (2007): congestion effects partially perform the role of an export tax.

The conjunction of this dynamic element in demand with the assumption of non-competitive market structure raises time-consistency issues that make optimal export policy prescriptions differ from the those derived for static models. To emphasize the importance of this time-consistency issue, this paper starts by analyzing the commitment equilibrium in the dynamic model, i.e. the equilibrium in which firms are assumed able to credibly commit to a path of output through time. The optimal policies in this case are identical to those derived in an analogous static analysis, both in the archetypal terms-of-trade exploitation case and the archetypal strategic trade case, and in the multi-firm case in which both motives coexist.

The paper then analyzes optimal policy in the presence of time-consistent firm behavior. Analyzed first is the case corresponding to the archetypal terms-of-trade exploitation case in which there is no strategic considerations concerning foreign output. Optimal policy in this time-consistent, i.e., Markov perfect, equilibrium generally calls for a non-zero tax or subsidy even when the domestic firm is a monopoly. If demand over time for the product is characterized by adjacent substitutability, the optimal policy is a tax, even if the industry is monopolized. If demand over time is characterized by adjacent complementarity, the optimal policy is either a subsidy, a tax, or neither, depending on the interplay of the number of firms and demand parameters.

Analyzed second is the case corresponding to the archetypal strategic trade case in which a sole domestic and a sole foreign firm compete as Nash-Cournot duopolists in a third country. Optimal policy in this Markov perfect equilibrium calls for a subsidy in the adjacent complementarity case, but may call for either a subsidy, a tax, or neither, depending on the interplay of cost and demand parameters, in the adjacent substitutability case.

As alluded to above, this paper interprets these results by showing that the assumption of time-consistent firm behavior in the presence of time-dependent demand and of a non-competitive market structure leads to steady-state equilibrium outcomes that mimic the commitment equilibrium of a different "competitive" situation, i.e., a situation with different numbers of domestic and foreign firms. That is, for given taste and technology parameters, the time-consistent steady state equilibrium is isomorphic to an equilibrium in which firms can commit to a path of output but in which the number of firms is different than what is specified in the time-consistent model. These commitment equilibria are essentially static equilibria, implying that the existing results of static models directly apply. Thus the optimal export policies in the dynamic time-consistent analyses correspond to standard static results of models which are analogous to the time-consistent models except for an assumption of a different number of firms.

The simplest possible environment was chosen in which to study the effects of the conjunction of time-non-separable preferences and non-competitive market structure. In particular, it is assumed that demand for the good is external to both the home and foreign country. For example, one might consider India and China exporting a good to the US market for which the domestic demand is nil. This study also abstracts from strategic interaction between the two governments, that is, the problem faced by a home country government assuming the foreign government is inert is analyzed. While these are potentially significant simplifications, the objective of this paper is to illuminate a non-transparent time-consistency issue that is operative in many dynamic settings with non-time-separable demand but would be obscured without the simplifications adopted.

2. The Model

2.1 Preliminaries

The basic model involves a two-stage process such as used by Driskill and McCafferty (1989), Dockner and Haug (1990), and Driskill and Horowitz (1996). At some time $t = 0$, the home-country government credibly sets a per-unit fixed export tax/subsidy of magnitude τ. If $\tau > 0$, this will be denoted as a tax, and if $\tau < 0$; this will be denoted as a subsidy. This tax or subsidy remains in place forever. The export firms then attempt to maximize the present discounted value of profits subject to this policy. In the commitment equilibrium, they do this by choosing a path of output for all time that is viewed as credible by consumers and rival firms. In the Markov perfect equilibrium (hereafter MPE), export firms choose state-dependent strategies.

The idea that τ remains fixed forever after $t = 0$ has some justification as an abstraction because the political processes in most countries do not permit quick, opportunistic policy changes. It is this inability to make quick opportunistic changes that in part makes the government's policy credible in the strategic trade analyses. An alternative approach is pursued by Leahy and Neary (1999), who in a two-period analysis allow, among other scenarios, commitment by the government on a period-by-period basis. While such an approach has the advantage of being richer in terms of potential government actions, it has the disadvantage of not permitting analysis of a steady state, and the results may thus be driven in part by initial conditions and end-game considerations. Moving beyond a two-period analysis with this approach is intractable. Regardless of whether the policy environment in which optimal export policies are analyzed is one in which terms-of-trade effects are assumed the only important consideration or one in which strategic trade effects dominate, it is assumed that all consumption is undertaken by residents of a third country, neither domestic or foreign. This simplification allows us to isolate the issues raised by the conjunction of dynamic demand and imperfect competition.

2.2 Third-country Demand

Instantaneous utility is a function of both current flow consumption of an "experience" good, denoted here by u, and the exponentially-weighted sum of past consumption of this good, denoted here by z; and also a non-experience good, denoted here by x. Assume a continuum of identical agents distributed over the unit interval. For notational convenience, consumer subscripts are omitted.

For each individual, "consumption capital" is given as a weighted average of past consumption of the experience good:

$$z(t) = \int_{-\infty}^{t} u(\varepsilon)e^{-s(t-\varepsilon)}d\varepsilon.$$

(1)

This implies

$$\dot{z} = u - sz$$

(2)

where s is a positive constant and the dotted z represents $\dfrac{dz}{dt}$. Following Driskill and McCafferty (2001), a quadratic instantaneous utility function that is also quasilinear with respect to the non-experience good is assumed:

$$v(t) = a_0 u - \frac{a}{2}u^2 + \beta_0 z - \frac{\beta}{2}z^2 + \delta uz + x. \tag{3}$$

For our purposes, it is assumed that $a_0 > 0$, $\alpha > 0$, $\beta_0 \geq$, $\beta > 0$, $\alpha\beta - \delta^2 > 0$, and $\delta \lessgtr 0$. An individual's budget constraint is given by

$$R = x + pu \tag{4}$$

where the price of x is normalized to 1 and p is the price of the experience good. In the infinite-horizon model, consumers choose u to maximize the present discounted value of instantaneous utility:

$$\max_u V = \int_0^\infty v(t)e^{-rt}dt \tag{5}$$

subject to (2) and (4). First-order conditions for this problem are

$$\frac{\partial L}{\partial u} = a_0 - \alpha u + \delta z - p + \lambda = 0 \tag{6}$$

$$-\frac{\partial L}{\partial z} = \dot{\lambda} - r\lambda = -\beta_0 + \beta z - \delta u + \lambda s \tag{7}$$

$$\lim_{T \to \infty} \lambda(T)e^{-rT} = 0 \tag{8}$$

where L is the current-value Hamiltonian, and λ is the current-value costate variable. Consumers have perfect foresight.

Depending on parameter values, this specification of consumer behavior encompasses a range of situations. For example, if $\delta = a_0 = 0$, this model can be interpreted as demand for a durable good subject to depreciation (measured by the parameter s) and adjustment costs of installation (measured by $\frac{a}{2} u^2$). If, on the other hand, the expression $\{\delta (r + 2s) - \beta\}$ is positive, this corresponds to what Becker and Murphy (1988) described as addictive behavior.

Some insight into the range of behaviors captured by this model of consumer behavior can be gathered by combining the consumer model with what might be called a benchmark model of supply. In this benchmark case, assume the supply of the experience good is generated by perfectly competitive firms producing under constant marginal cost conditions, so that $p = c_0$, where c_0 is a constant. Time-differentiating equation (6), noting that $\dot{p} = 0$, and combining this resulting equation with equations (6), (7), and (2) yields the following coupled system of differential equations in u and z:

$$\dot{u} = (r+s)u - \frac{1}{\alpha}Az - \frac{1}{\alpha}\{a_0(r+s) + \beta_0 - c_0(r+s)\}; \tag{9}$$

$$\dot{z} = u - sz \tag{10}$$

where $A \equiv \{\delta(r + 2s) - \beta\}$. This composite parameter "A" is negative if $\delta < 0$, but can be positive if δ is positive and if $\delta(r + 2s) > \beta$. As noted, Becker and Murphy (1988) characterized preferences for a good in which $A > 0$ as addictive. Assuming $\{\alpha_0(r + s) + \beta_0 - c_0(r + s)\} > 0$, the condition for a non-negative steady-state value of u (and, of course, z) is $A < \alpha s(r + s)$. Assuming these conditions hold, the stable arm of the saddlepath for this coupled system is calculated as:

$$u = \gamma_0 - \gamma z \tag{11}$$

where

$$\gamma = \frac{1}{2}[r + 2s - \sqrt{(r + 2s)^2 - 4\frac{A}{\alpha}}] \tag{12}$$

$$\gamma_0 = \frac{\frac{1}{\alpha}\{\alpha_0(r + s) + \beta_0 - c_0(r + s)\}}{\gamma - (r + s)} \tag{13}$$

The key feature of this "decision rule" is that the slope, given by γ, is positive if $A > 0$, negative if $A < 0$, and zero if $A = 0$. Thus, for individuals who rationally expect a constant price through time, higher levels of past consumption, i.e., higher values of z, imply higher or lower levels of current consumption as $A \gtrless 0$. This feature of consumer behavior in this benchmark model presumably motivated Becker and Murphy (1988) to equate "addictive" behavior with preference parameter values such that $A > 0$. They also pointed out that addictive behavior could be said to be synonymous with *adjacent complementarity* of the good over adjacent time periods. Non-addictive behavior would then be synonymous with *adjacent substitutability*.

With imperfect competition, the interactions between supply and demand are more complicated than in this case of perfect competition, and depend on assumptions about whether firms can commit to paths of output or whether they must choose time-consistent, i.e., Markov, strategies. This means that in an MPE, the slope parameter in consumers' decision rules, that is, γ, is a more complicated function of underlying taste and cost parameters and of the number of firms in the industry. The algebraic sign of A, though, will remain an important feature of the solutions in both the commitment and Markov perfect equilibria. Consequently, the same language used in the perfect-competition constant-marginal-cost case is retained, and the situation exhibiting *adjacent complementarity* or *adjacent substitutability* depending on whether $A \gtrless 0$, respectively, is described.

It is also useful to point out the steady-state relationship between price and output that is found by setting all time-derivatives to zero in (6) and (7) and rearranging terms:

$$\bar{p} = \alpha_0 + \frac{\beta_0}{r + s} + \bar{u}\left[\frac{A}{s(r + s)} - \alpha\right] \tag{14}$$

3. The Commitment Equilibrium and the Optimal Policy

In a commitment equilibrium, firms are assumed to credibly commit to a path of output through time. Following Driskill and McCafferty (2001), the possibility of multi-plant firms and rising marginal costs of production is allowed for. In particular, assume there are n identical domestic firms, each indexed by i, and n^* identical foreign firms, indexed by i^*.

Consider the domestic firm analysis first. Each domestic firm owns $1/n$ of M identical production units, each unit indexed by m and having instantaneous cost of production given by

$$C_m = F_m + c_0 u_m + \frac{\hat{c}}{2} u_m^2; \, m = 1, 2, \ldots, M, \tag{15}$$

where F_m denotes plant fixed costs, u_m is plant output, and c_0 and $\hat{c}$ are non-negative constants. Assuming firms minimize costs by producing equal amounts at each plant, firm cost of production will be given by

$$C_i = c_0 u_i + \frac{nc}{2} u_i^2 + F_i; \, c = \frac{\hat{c}}{M}, \, i = 1, 2, \ldots, n \tag{16}$$

where u_i is the ith firm output and F_i are firm fixed costs. This assumption about cost functions leaves the *industry* cost function invariant to the number of firms within the industry.[5] Note that for simplicity integer constraints are ignored and the number of plants and number of firms are taken as being exogenous.

By analogous steps the i^{*th} foreign firm's cost function are derived as

$$C_{i^*} = c_0 u_{i^*} + \frac{n^* c^*}{2} u_{i^*}^2 + F_{i^*}; c^* = \frac{\hat{c}^*}{M^*}, \, i^* = 1, 2, \ldots, n^* \tag{17}$$

Firms take as given other firms' strategies and the optimal behavior of consumers as embodied in the first-order conditions (6) and (7). The ith firm's problem is to choose a strategy $u_i(t)$ so as to maximize the present discounted value of profits:

$$\max_{u_i \in S} \Pi_i \equiv \int_0^\infty \{p(\lambda, u, z) u_i - (c_0 + \tau) u_i - \frac{nc}{2} u_i^2\} e^{-rt} \, dt \tag{18}$$

$$s.t. \, \dot{z} = u - sz \tag{19}$$

$$p = \lambda + \alpha_0 - \alpha u + \delta z \tag{20}$$

$$\dot{\lambda} = \lambda(r + s) - \beta_0 + \beta z - \delta u \tag{21}$$

where $u \equiv \sum_{i=1}^{n} u_i + \sum_{i^*=1}^{n^*} u_{i^*}$ and S_i, the firm's strategy space, is defined as $S_i = \{u_i(t)$ such that $u_i(t)$ is continuous and differentiable$\}$.

---

[5] This feature means that a merger, for example, that would simply change the number of firms but not the number of plants, would not change the industry cost function.

First-order conditions are:

$$\lambda + a_0 - \alpha u + \delta z - \alpha u_i - (c_0 + \tau) - ncu_i + \theta_i - \delta\phi_i = 0 \tag{22}$$

$$\dot{\theta}_i = (r+s)\theta_i - \delta u_i - \beta\phi_i \tag{23}$$

$$\dot{\phi}_i = -s\phi_i - u_i. \tag{24}$$

Analogous first-order conditions obtain for foreign firms. Because this commitment equilibrium is primarily of interest as a benchmark, the dynamic analysis is sidestepped and just the steady state is analyzed.[6] Denote steady-state output of the foreign industry as u_F and steady-state output of the home industry as u_H. Total steady-state output, sold in the third-country market, is denoted as usual by $\bar{u}$. That is, $\bar{u} = u_F + u_H$.

From the consumer problem, the steady-state value of λ is a function of $\bar{u}$:

$$\bar{\lambda} = \frac{\beta_0}{r+s} - \frac{(\beta - \delta s)}{s(r+s)}\bar{u} \tag{25}$$

Combining this with the first-order conditions for each (identical) firm in the foreign country, in the steady state, gives the foreign country's aggregate reaction function:

$$u_F = \frac{a - c^* - bu_H}{c^* + b\left(1 + \frac{1}{n^*}\right)} \tag{26}$$

where, for notational ease, the following substitutions have been introduced:

$$a \equiv \frac{\beta_0}{r+s} + \alpha_0 \tag{27}$$

and

$$b \equiv \alpha - \frac{A}{s(r+s)}. \tag{28}$$

Throughout the paper it is assumed that $b > 0$. Finally, note that the steady-state price as a function of $\bar{u}$ is

$$\bar{p} = a - b(u_H + u_F) \tag{29}$$

Welfare of the home country is measured by the value of steady-state profits plus the export tax revenues:

$$W = \bar{p}u_H - c_0 u_H - \frac{c}{2}u_H^2 \tag{30}$$

---

[6] Because of the dimension of the two-country problem, analysis of the dynamic paths prove to be computationally challenging. The insights gained from the steady state analysis are what is most important here, so the complete dynamic analysis is not attempted. Details of the dynamic analysis for a single country are available from the authors.

Note that tax revenues do not show up in the above expression because they are subtracted from firm profits and added back as a component of welfare.

First note that the welfare-maximizing level of u_H can be computed by choosing u_H to maximize

$$W = \bar{p}(u_H + u_F(u_H))u_H - c_0 u_H - \frac{c}{2}(u_H)^2. \tag{31}$$

This value of u_H, denoted as u_H^{max}, is

$$u_H^{max} = \frac{a(\frac{b}{n^*}+c^*)+bc_0^* - b(1+\frac{1}{n^*}) - c_0 c_0^*}{\frac{2b^2}{n^*}+2bc^*+cb(1+\frac{1}{n^*})+cc^*}. \tag{32}$$

Now, using the steady-state first-order conditions for the (identical) home firms and the steady-state conditions for the consumer problem and the foreign country reaction function, the equilibrium value of u_H can be computed:

$$\hat{u}_H = \frac{G - \tau[c^* + b(1+\frac{1}{n^*})]}{b[c(1+\frac{1}{n^*})+c^*(1+\frac{1}{n})]+b^2[\frac{1}{n}+\frac{1}{n^*}+\frac{1}{nn^*}]+cc^*} \tag{33}$$

where $G \equiv [a(\frac{b}{n^*}+c^*)+b(c_0^* - (1+\frac{1}{n^*})c_0) - c_0 c^*]$. It is assumed that $G > 0$; so that $\hat{u}_H > 0$ when $\tau = 0$:

Equating u_H^{max} to $\hat{u}_H$ yields the optimal tax:

$$\tilde{\tau}c = \frac{n^*G\left\{b\left[b\left\{\frac{n-(n^*+1)}{n}\right\}+n^*c^*(1-\frac{1}{n})\right]\right\}}{[2b^2+bn^*\{2c^*+c(1+\frac{1}{n^*})\}+n^*cc^*][n^*c^*+b(n^*+1)]} \tag{34}$$

Whether the optimal policy is a tax ($\tilde{\tau}_c > 0$) or a subsidy ($\tilde{\tau}_c < 0$) depends on the sign of the expression:

$$b\left\{\frac{n-(n^*+1)}{n}\right\}+n^*c^*(1-\frac{1}{n})$$

If this is positive, the optimal policy is a tax, if it is negative, the optimal policy is a subsidy, and if it is zero the optimal policy is laissez-faire.

To understand the interplay of the components that make up this expression, first consider the expression in the left-side bracket. With relatively small numbers of domestic firms, the profit-shifting motive dominates the terms-of-trade motive and the expression is negative. With relatively large numbers of domestic firms, the terms-of-trade gains from cartelization of the domestic industry are more important and the expression is positive.[7]

---

[7] Dixit (1984), who assumed constant marginal cost, derives the same condition as done here in the constant marginal cost case: the optimal export policy is a tax only if the number of domestic firms exceeds the number of foreign firms plus 1.

The right-side expression captures only terms-of-trade/cartelization effects. When $n = 1$, there is obviously no scope to cartelize domestic output via an export tax, but there is with $n > 1$. The presence of the foreign marginal cost parameter also reflects terms-of-trade effects: restriction of home-country output by an export tax has more of an effect on price the more foreign firm responses to this diminished output are dampened by increasing marginal cost.

To further understand this optimal policy formula, the archetypal case in which the terms-of-trade motive acts alone in influencing optimal policy is considered. That is, the case of no foreign firms, i.e., $n^* = 0$ is considered. In this case,

$$\tilde{t}_C (n^* = 0) = \frac{b(a - c_0) \left[1 - \frac{1}{n}\right]}{2b + c} \tag{35}$$

The implications of this optimal policy formula can be summarized in the following proposition, the proofs of which are immediate:

Proposition 1 *In the commitment equilibrium in which there are no foreign firms, the optimal export policy is a tax ($\hat{t}_C > 0$) if there is more than one domestic firm and is laissez-faire ($\hat{t}_C = 0$) if there is only one domestic firm.*

That is, the optimal export policy is a tax that induces the domestic industry to behave as a monopolist.

Now consider the expression when $n = n^* = 1$, the classic duopoly case studied in strategic trade analysis:

$$\tilde{t}_C (n = n^* = 1) = \frac{G \left[-b^2\right]}{(c^* + 2b) \left[2b^2 + 2b(c + c^*) + cc^*\right]} \tag{36}$$

The implication of this expression can be summarized as:

Proposition 2 *In the commitment equilibrium when there is one domestic and one foreign firm competing in a third-country market, the optimal export policy of the home country is an export subsidy.*

Given that the commitment equilibrium is analogous to a one-shot game, it is not surprising that the standard duopoly result of the optimal policy being a subsidy obtains here. Note that marginal cost assumptions in this duopoly case make no difference to the result: with a monopolist in the home country there is no scope for terms-of-trade gains, regardless of marginal cost.

Finally of interest to us is the case where $n = n^* = N$, i.e., the case of equal numbers of domestic and foreign firms. In this case,

$$\tilde{t}_C (n = n^* = N) = \frac{Gb\left[\frac{-b}{N} + c^*(1 - \frac{1}{N})\right]}{c^* + b(1 + \frac{1}{N})} \tag{37}$$

Two aspects of this last optimal tax formula will be made use of. First, when $c^* = 0$, the optimal policy is always a subsidy, no matter the size of N. This subsidy, though, tends to zero as $N \to \infty$. Second, when $c^* > 0$, there exist a critical number of firms below which the optimal policy is a subsidy and above which the optimal policy is a tax.

Again, this is perhaps not surprising in that one would expect the profit-shifting motive of strategic trade policy to disappear in the presence of more competitive-like environments. These implications are summarized in the following proposition:

Proposition 3 *In the commitment equilibrium with equal numbers (greater than 1) of domestic and foreign firms:*

1. *If foreign marginal cost is constant, the optimal export policy is a subsidy;*
2. *If foreign marginal cost is increasing, then there exists a critical number of firms, $\tilde{N}(b, c^*)$, such that for $N < \tilde{N}$ the optimal export policy is a subsidy and for $N > \tilde{N}$ the optimal policy is tax.*

    Proof. *The first part is obvious from inspection of the optimal tax expression. To prove the second part, define the function $\phi(N) = -\frac{1}{N}(b + c^*) + c^*$. This is a monotonically increasing continuous function of N with $\phi(1) = -b < 0$, $\phi' > 0$; and $\phi(\infty) = c^* > 0$. Hence, there exists a value of N, call it $\tilde{N}$, for which $\phi(\tilde{N}) = 0$.*

The intuition here comes from thinking about the two motives. With constant marginal cost and equal numbers of domestic and foreign firms, the terms-of-trade motive for a tax gets no help from the dampening effect on foreign output increases of increasing foreign marginal cost. And with increasing foreign marginal cost, a "more competitive" world market diminishes the profit-shifting motive for an export tax.

This paper will now consider the more interesting and relevant analyses of perfect Markov equilibria. The strategy here, though, is to show how the steady states of these equilibria can be thought of as more-or-less competitive analogs of analogous static models. The intuition behind the optimal policies in these truly dynamic settings can thus be derived from the preceding analysis of commitment equilibria.

First to be analyzed is the MPE of the archetypal terms-of-trade case in which there are no foreign firms. This highlights that the key issue is commitment. The archetypal strategic trade model of a single domestic and single foreign firm are then analyzed.

4. The Markov Perfect Equilibrium in the Terms-of-Trade Exploitation Case with No Foreign Firms

4.1 Demand

When firms cannot commit to a path for output, and consumers know this, then they know that the equilibrium values of u are a linear function of the state, z:

$$u(t) = \gamma_0 + \gamma z(t) \tag{38}$$

where γ_0 and γ are as-yet-to-be-determined parameters. Knowing this relationship between u and γ, and integrating (7) from t to ∞ and substituting the resulting expression into (6) yields the consumers' decision rule. This rule is the instantaneous demand curve that constrains firms at any moment in time:

$$p(t) = \chi_0 - \alpha u + \chi z \tag{39}$$

where

$$\chi_0 = \frac{\chi \gamma_0 + (r+s)\alpha_0 - \beta_0}{r+s} \tag{40}$$

$$\chi = \frac{A}{r + 2s - \gamma} \equiv \psi(\gamma) \tag{41}$$

and, again, $A \equiv \delta(r + 2s) - \beta$.[8]

Equation (40) describes a demand curve that is downward-sloping at any moment but that shifts as z changes. Whether it shifts in or out for an increase in z depends on the sign of χ, which is in equilibrium a function of all the structural parameters of the model. Note that if $\chi = 0$, this is a standard static demand curve.

Equations (41; 42) are a pair of relationships between (χ, γ) and (χ_0, γ_0). Note (42) links (χ, γ), but is independent of (χ_0, γ_0). Firm behavior will provide another pair of relationships between these pairs of variables. These four relationships will then determine the equilibrium values of (χ, γ) and (χ_0, γ_0) as functions of structural parameters.

4.2 Firm Behavior

Firms are assumed to pick strategies from a Markov strategy space. These strategies describe decision rules that are a function of the state, z, and time, t. The first-order conditions for the ith firm are thus

$$p = \alpha u_i + (c_0 + \tau) + ncu_i - \lambda_i \tag{42}$$

$$\dot{\lambda}_i = \lambda_i \left[r + s - \sum_{j \neq i} \frac{\partial u_j(z,t)}{\partial z} \right]$$

$$-\chi u_i + \alpha u_i \sum_{j \neq i} \frac{\partial u_j(z,t)}{\partial z}. \tag{43}$$

Assume the firms' strategies are symmetric and a linear function of z:

$$u_i(z, t) = m_0 + mz(t); \ m_0 \in R; \ m \in R . \tag{44}$$

With this assumption, time-differentiating (43), substituting this result into (44) and substituting (43) into this expression for λ_i does in fact yield a linear relationship between u_i and z. This is the firm's strategy. Aggregating this strategy over all firms yields a linear relationship between u and z. Equating coefficients between this relationship and (42) yields the promised second pair of relationships between (χ, γ) and (χ_0, γ_0):

---

[8] Further details of this derivation can be found in Driskill and McCafferty (2001). It is also shown there that this demand curve can be derived as the limit of a finite horizon problem as the horizon goes to infinity.

$$\chi = \frac{\gamma[\alpha(1+\frac{1}{n})+c](r+2s)}{r+2s-2\gamma(1-\frac{1}{n})}$$

$$-\frac{\gamma^2[2\alpha+(2-\frac{1}{n})c]}{r+2s-2\gamma(1-\frac{1}{n})}$$

$$\equiv \phi(\gamma);$$
(45)

$$-\frac{\{r+s-(1-\frac{1}{n})\gamma\}(c_0-\chi_0)}{\{r+s-(2-\frac{1}{n})\gamma\}}(c_0-\chi_0) = \gamma_0\{(1+\frac{1}{n})\alpha+c\}$$

$$+\frac{\chi(1-\frac{1}{n})+\alpha\gamma(\frac{n-1}{n^2})}{\{r+s-(2-\frac{1}{n})\gamma\}}$$
(46)

Once the value of γ that simultaneously solves (46) and (42) is found, it can be used to solve recursively all other endogenous variables in the model, in particular γ_0 and χ. Knowledge of the values of these endogenous parameters allows one to describe the equilibrium strategies of the two firms and the equilibrium decision rule of the consumers. Note that, because $\dot{z} = u - sz$, asymptotic stability of the model would require that $\gamma < s$.

In Driskill and McCafferty (2001), the following proposition that describes the MPE is proved:

Proposition 4 *Assume* $\frac{A}{s(r+s)} < \alpha$ *and* $\alpha_0 + \frac{\beta_0}{r+s} - c_0 - \tau > 0$. *Then the unique stable MPE of the preceding game has the following steady state value of u:*

$$\bar{u}_{MPE}(n) = \frac{\alpha_0 + \frac{\beta_0}{r+s} - (c_0+\tau)}{\alpha+c - \frac{A}{s(r+s)} + \frac{1}{n}\left\{\frac{\alpha(r+s)-\chi}{r+s-(1-\frac{1}{n})\gamma}\right\}}.$$
(47)

Furthermore, $\chi \lesseqgtr 0$ *and* $\gamma \lesseqgtr 0$ *as* $A \lesseqgtr 0$. *In addition,* $(\alpha(r+s)-\chi) > 0$.

A key feature of the above expression for $\bar{u}_{MPE}(n)$ is that $\frac{\partial \bar{u}_{MPE}(n)}{\partial \tau} < 0$. This is only obvious because the endogenous parameters γ and χ do not depend on τ.

4.3 The Optimal Policy

Using the expression in equation (14) for the steady-state value of p, one can find the value of $\bar{u}$ that maximizes steady state welfare, $W = \bar{p}\bar{u} - c_0\bar{u} - \frac{c}{2}(\bar{u})^2$:

$$\bar{u}_{WM} = \frac{1}{2}\left[\frac{\alpha_0 + \frac{\beta_0}{r+s} - c_0}{\alpha+\frac{c}{2} - \frac{A}{s(r+s)}}\right].$$
(48)

Equating this value of $\bar{u}$ to $\bar{u}_{MPE}(n)$ and solving for τ yields the optimal tax/subsidy:

$$\hat{\tau}_{MPE}(n) = \left[\frac{\alpha_0 + \frac{\beta_0}{r+s} - c_0}{2\alpha+c - \frac{2A}{s(r+s)}}\right]\left[\alpha - \frac{A}{s(r+s)} - \frac{1}{n}\left\{\frac{\alpha(r+s)-\chi}{r+s-(1-\frac{1}{n})\gamma}\right\}\right]$$
(49)

Because it is assumed that $\{\alpha + c - \frac{A}{s(r+s)}\} > 0$ and $\{\alpha_0 + \frac{\beta_0}{r+s} - c_0\} > 0$, whether the optimal policy is a tax ($\tau > 0$) or a subsidy ($\tau < 0$) depends on whether $\{\alpha - \frac{A}{s(r+s)}\} \gtrless \frac{1}{n} \left\{\frac{\alpha(r+s) - \chi}{r+s - (1 - \frac{1}{n})\gamma}\right\}$. Because γ and χ are endogenous, the relationship between these two quantities is not obvious. But it is straightforward to show that: (1) If $A < 0$, then $\{\alpha - \frac{A}{s(r+s)}\} > \frac{1}{n} \left\{\frac{\alpha(r+s) - \chi}{r+s - (1 - \frac{1}{n})\gamma}\right\}$; (2) If $A = 0$, $\{\alpha - \frac{A}{s(r+s)}\} > \frac{1}{n} \left\{\frac{\alpha(r+s) - \chi}{r+s - (1 - \frac{1}{n})\gamma}\right\}$; (3) If $A \in (0, \alpha s(r + s))$, then $\{\alpha - \frac{A}{s(r+s)}\} > \left\{\frac{\alpha(r+s) - \chi}{r+s - (1 - \frac{1}{n})\gamma}\right\}$, and $\bar{u}_C(n) > \bar{u}_{MPE}(n)$; (4) If $A \in (0, \alpha s(r + s))$, then there exists an $n > 1$ such that $\{\alpha - \frac{A}{s(r+s)}\} = \frac{1}{n} \left\{\frac{\alpha(r+s) - \chi}{r+s - (1 - \frac{1}{n})\gamma}\right\}$.

Thus, for various ranges of A, there is the following proposition that describes the qualitative properties of the optimal policy:

Proposition 5 *In the Markov perfect equilibrium without foreign production:*

1. *If $A < 0$; then the optimal policy is a tax, regardless of the number of firms, i.e., $\hat{\tau}_{MPE}(n) > 0 \ \forall n; n = 1, 2, \ldots$;*
2. *If $A = 0$, then $\hat{\tau}_{MPE}(1) = 0$ and $\hat{\tau}_{MPE}(n) > 0 \ \forall n; n = 1, 2, \ldots$;*
3. *If $A \in (0, \alpha s(r + s))$, then:*
   (a) *If the home country industry is a monopoly, the optimal policy is a subsidy, i.e., $\hat{\tau}_{MPE}(1) < 0$;*
   (b) *If the home country industry is a Nash-Cournot oligopoly, the optimal policy is a subsidy for a sufficiently small number of firms and a tax for sufficiently large number of firms, i.e., there exists some finite values of $n > 1$ such that $\hat{\tau}_{MPE}(n) > 0$:*

The contrast between the dynamic time-consistent analysis and the static or commitment analysis is perhaps seen most sharply in the results on monopoly. For a monopolized domestic industry either facing a static demand curve or facing the dynamic demand of this analysis but having the ability to commit to a time path of output, the optimal policy is a zero tax, i.e., laissez-faire. In the MPE, though, the optimal policy is a zero tax *only* when $A = 0$. When $A = 0$, the demand curve constraining firms at every moment is in fact static, i.e., depends only on current u, and the optimal policy in the MPE should be expected to mimic the traditional static analysis. But as long as $A \neq 0$, the optimal policy in the MPE is not a zero tax. Dynamics only matter when firms cannot commit.

The other parts of the proposition can be interpreted in light of the relationship between the steady state value of output in the MPE with no foreign firms, $\bar{u}_{MPE}(n)$, and the steady state value of output in the commitment equilibrium with no foreign firms, which will be denoted by $\bar{u}_C(n)$. From equation (33), a straightforward computation yields

$$\bar{u}_C(n) = \frac{\alpha_0 + \frac{\beta_0}{r+s} - (c_0 + \tau)}{\alpha + c - \frac{A}{s(r+s)} + \frac{1}{n}\left\{\alpha - \frac{A}{s(r+s)}\right\}}$$

From inspection of the equations for $\bar{u}_{MPE}(n)$ and $\bar{u}_C(n)$, it is clear that

$$\bar{u}_C(n) \lessgtr \bar{u}_{MPE}(n) \text{ as } \left\{\alpha - \frac{A}{s(r+s)}\right\} \gtrless \left\{\frac{\alpha(r+s)-\chi}{r+s-(1-\frac{1}{n})\gamma}\right\}. \text{ As noted, if } A < 0, \text{ then}$$

$\left\{\alpha - \frac{A}{s(r+s)}\right\} > \left\{\frac{\alpha(r+s)-\chi}{r+s-(1-\frac{1}{n})\gamma}\right\}$, and thus $\bar{u}_C(n) < \bar{u}_{MPE}(n)$. Also as noted, if $A \in$

$(0, \alpha s(r + s))$, then $\left\{\alpha - \frac{A}{s(r+s)}\right\} > \left\{\frac{\alpha(r+s)-\chi}{r+s-(1-\frac{1}{n})\gamma}\right\}$, and $\bar{u}_C(n) > \bar{u}_{MPE}(n)$. That is,

for $A < 0$, the steady state Markov perfect equilibrium value of output is pro-competitive vis-à-vis the steady-state commitment equilibrium value of output, and for $A \in (0, \alpha s(r + s))$, the reverse is true. Furthermore, $\lim_{n\to\infty}\bar{u}_C(n) = \lim_{n\to\infty}\bar{u}_{MPE}(n)$. This can be summarized in the following proposition:

Proposition 6 *If preferences exhibit adjacent substitutability (complementarity), i.e., A < 0 (A > 0), then the steady-state level of output in the commitment equilibrium is less than (greater than) the steady-state level of output in the MPE equilibrium. That is, if A < 0; $\bar{u}_C(n) < \bar{u}_{MPE}(n)$; and if A > 0; $\bar{u}_C(n) > \bar{u}_{MPE}(n)$. Furthermore, $\lim_{n\to\infty}\bar{u}_C(n) = \lim_{n\to\infty}\bar{u}_{MPE}(n)$.*

This is depicted in Figure 1. For the case of $A < 0$, the solid curve represents $\bar{u}_C(n)$ and the dotted curve represents $\bar{u}_{MPE}(n)$. For $A > 0$, the solid curve would represent $\bar{u}_{MPE}(n)$ and the dotted curve $\bar{u}_C(n)$: The horizontal dashed line shows that if $A < 0$ there is a value of n such that $\bar{u}_{MPE}(n) = \bar{u}_C(1)$. If $A > 0$, this line is interpreted as depicting that value of n above which $\bar{u}_{MPE}(n) > \bar{u}_C(1)$.

Figure 1: A comparison between $\bar{u}_C(n)$ and $\bar{u}_{MPE}(n)$
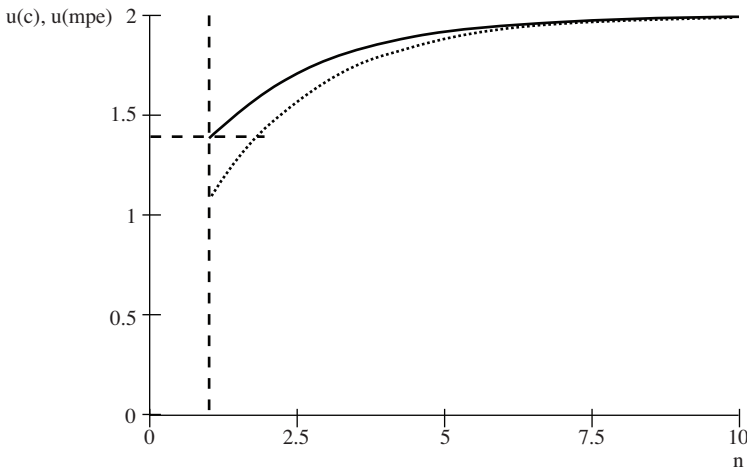
As a prelude to interpreting the optimal policy results, the policy results for the commitment equilibrium will be reviewed. In that analysis, the optimal policy is to use τ to get the steady-state level of output equal to the commitment equilibrium monopoly level, $\bar{u}_C(1)$. This requires a tax as long as $A < \alpha s(r + s)$ and $n > 1$. This is because, in this

case, $\frac{\partial \bar{u}_C(n)}{\partial n} > 0$ so steady state output is above the optimal level for any $n > 1$. Conse-quently, to achieve the optimal output level requires a tax that causes each individual firm to reduce their output $(\frac{\partial \bar{u}_C(n)}{\partial \tau} < 0)$.

Now, for $A < 0$, in the Markov perfect equilibrium the optimum policy requires a tax a fortiori because $\bar{u}_C(n) < \bar{u}_{MPE}(n)$. That is, output in the steady state of the MPE for any given n, say $\hat{n}$; is isomorphic to output in the steady state of the commitment equilibrium for some $n < \hat{n}$.

For $A \in (0, \alpha s(r + s))$, though, $\bar{u}_C(n) > \bar{u}_{MPE}(n)$. For $n = 1$, this clearly implies that the optimal MPE policy is a subsidy that increases $\bar{u}_{MPE}(1)$. For large enough values of n, though, $\bar{u}_{MPE}(n) > \bar{u}_C(1) = \bar{u}^*$, and the optimum policy is a tax that decreases $\bar{u}_{MPE}(n)$.

5. Strategic Trade Analysis

This paper will now solve for the optimal tax/subsidy policy in the MPE when there is only one domestic and one foreign firm. This time-consistent result will be interpreted by comparing it with the multiple-firm commitment-equilibrium results.

5.1 The Markov Perfect Equilibrium and Optimal Policy

Solving the Markov perfect equilibrium, will be restricted to the analysis of two duopolists, one home and one foreign. Output of the home-country firm is denoted by u_h and output of the foreign-country rival by u_f; and steady-state outputs by u_H and u_F, respectively. Cost functions are identical, and are given by:[9]

$$C_h = \frac{c(u_h)^2}{2};$$

$$C_f = \frac{c(u_f)^2}{2}$$

The third-country demand curve constraining these firms is the same as in the analysis of the Markov perfect equilibrium in the case of only a home-country industry, i.e., equation (38). That is, consumers know that current output is a linear function of the state and know that firms are time-consistent optimizers. Firms choose Markov strategies. The firm's problem is thus to choose a strategy $u_i(z); i = h; f$, so as to maximize the present discounted value of profits, J_i; given as:

$$J_i = \int_0^\infty \{p(u, z)u_i - \tau_i u_i - \frac{c}{2}(u_i)^2\}e^{-rt} dt, \ i = h, f, \tag{50}$$

subject to:

---

[9] This has been simplified by assuming the intercept terms of the cost functions are 0 for simplicity: the calculations involved in deriving the optimal tax/subsidy are greatly simplified by this assumption.

$$\dot{z} = u_h + u_f - sz; \tag{51}$$

$$p = \chi_0 - \alpha u + \chi z; \tag{52}$$

$$u = u_h + u_f \tag{53}$$

where τ_i is, as before, a tax if positive and a subsidy if negative. The only difference between this analysis and that of the previous analysis of a Markov perfect equilibrium is that the equilibrium firm strategies in this case are asymmetric in terms of their intercepts. That is, using the same techniques as before,[10] the Markov perfect equilibrium of this game can be characterized as pairs of firm strategies and a consumer decision rule:

    Firm strategies:

$$u_i = K_i + kz, i = h.f; \; K_h + K_f = \gamma_0; \; k = \frac{\gamma}{2}. \tag{54}$$

    Consumer decision rule:

$$p = \chi_0 - \alpha u + \chi z \tag{55}$$

where the equilibrium value of γ; denoted by $\hat{\gamma}$, is the unique solution to

$$\gamma < s; \tag{56}$$

$$(\gamma)^2 [2\alpha + \frac{3}{2}c] - \gamma[(r + 2s)(\frac{3}{2}\alpha + c) + A \;\; = \;\; 0 \tag{57}$$

and the equilibrium value of χ, denoted by $\hat{\chi}$ is given by

$$\hat{\chi} = \frac{A}{r + 2s - \hat{\gamma}}. \tag{58}$$

    The values of χ_0, K_h and K_f are linear functions of the equilibrium values of γ and χ and the export tax parameter τ.

    Of most interest are the steady-state equilibrium values:

$$u_H = \frac{a}{2\Delta} + \tau \left[\frac{-b - 2(c + \frac{\omega}{2})}{4\Delta(c + \frac{\omega}{2})} \right] \tag{59}$$

$$\bar{u} = \frac{a - \frac{\tau}{2}}{\Delta} \tag{60}$$

$$\bar{p} = a \left[\frac{\Delta - b}{\Delta} \right] + \tau \left[\frac{b}{2\Delta} \right] \tag{61}$$

---

[10] Again, see Driskill and McCafferty (2001) for more details.

where

$$\Delta \equiv c + b + \frac{1}{2}\omega; \tag{62}$$

$$\omega \equiv \left\{ \frac{\alpha(r+s) - \chi}{r + s - \frac{\gamma}{2}} \right\} \tag{63}$$

and a and b are as defined in (27) and (28), respectively. It is assumed that $\Delta > 0$ so that steady-state values are positive. With these steady-state values, the optimal export tax formula can be found:

$$\hat{\tau} = \frac{\left(\frac{a}{2\Delta^2}\right)\left[-\frac{\omega^2}{4} + c(b-\omega)\right]}{\left[\frac{b\{b+2(c+\frac{\omega}{2})\}}{2\Delta^2(c+\frac{\omega}{2})} + c\left\{\frac{\{b+2(c+\frac{\omega}{2})\}^2}{2\Delta^2(c+\frac{\omega}{2})^2}\right\}\right]}. \tag{64}$$

For $A < \alpha s(r + s)$, $b > 0$ (by definition), and it is straightforward to show that $c + \frac{\omega}{2} > 0$: Hence, it is immediately clear that the optimal export policy is a subsidy, i.e., $\hat{\tau} < 0$, when there is no increasing marginal cost, i.e., when $c = 0$.

When $c > 0$, though, it is only certain that the optimal export policy is a subsidy when $b < \omega$. It is straightforward to show that $b > \omega$ if $A < 0$, and $b < \omega$ if $A \in (0, \alpha s(r + s)]$: Hence, the following proposition:

Proposition 7 *Assume $A < \alpha s(r + s)$): Then:*

1. *If $c = 0$, the optimal export policy is a subsidy, i.e., $\hat{\tau} < 0$.*
2. *If $A \in (0; \alpha s(r + s)]$, the optimal export policy is a subsidy regardless of the value of c; i.e., $\hat{\tau} < 0$.*
3. *If $A < 0$; there exist parameter values for which the optimal export policy is a tax, i.e., for which $\hat{\tau} > 0$.*

While a general characterization of parameter values for which $\hat{\tau} > 0$ is extremely complicated, simulations with specific parameter values generate cases in which $\hat{\tau} > 0$. For example, $r = 0$; $s = 7$; $\alpha = c = 1$; and $A = -84$ yields: $\gamma = -2$, $\chi = 5.25$, $\omega = \frac{49}{32}$, $b = \frac{19}{7}$, and

$$\frac{-\omega^2}{4} + c(b - \omega) \approx .59.$$

Furthermore, we know from Driskill and Horowitz (1996) that for the special case of $\delta = \alpha = 0$ (the case of a durable good without installation adjustment costs), $\hat{\tau} > 0$.

Intuition about this result comes from a comparison with the commitment equilibrium case in which there are equal numbers of domestic and foreign firms, i.e., in which $N = N^*$. There, as here, the optimal policy was always a subsidy when the foreign marginal cost was constant, i.e., $c^* = 0$: the terms-of-trade/cartelization motive is thwarted by the elastic response of foreign firms to a reduction in domestic output. And there, the optimal policy was more likely to be a tax the more competitive was the environment, i.e., the larger the value of N, the number of firms in each country. Here, $A < 0$ corresponds to

a "more competitive" environment that is analogous to a larger value of N, and there is also the greater likelihood of the optimal policy being a tax, as long as there is increasing marginal cost. In contrast, $A > 0$ corresponds to a "less competitive" environment, and the optimal policy is a subsidy.

Finally, note the contrast with the time-separable case in which $A = 0$. In that case, $b = \omega = \alpha$, and the optimal policy is always a subsidy, never a tax, regardless of the assumption about marginal cost. This corresponds to the archetypal Brander-Spencer example, and with only one home firm, there is no scope for terms-of-trade exploitation.

6. Conclusions

The conjunction of dynamic demand and monopoly or oligopoly market structure leads to time-consistency issues that change the usual optimal export policy prescriptions vis-à-vis those derived from the standard static analyses. The importance of the time-consistency issue is most sharply illustrated in the case of a domestic monopoly facing third-country demand and no foreign supply. In this situation, the optimal policy prescription is a zero export tax in both the static model and in the dynamic model in which firms are assumed to be able to commit to paths of output through all future time. In contrast, the policy prescription is either a tax or subsidy, depending on preference and cost parameters, in the dynamic model in which firms cannot commit, i.e., in which the equilibrium is Markov perfect.

This paper also generates optimal policy prescriptions in the archetypal strategic trade case of two duopolists, one home and one foreign, who compete in a third-country market. In static Nash-Cournot analyses of this situation, the policy prescription is the standard Brander-Spencer export subsidy. In a commitment equilibrium, this remains the case. But in the Markov perfect equilibrium, the optimal policy may be a tax if production faces increasing marginal cost and if demand is characterized by adjacent substitutability (which subsumes durability). Again, the time consistency issue, not dynamics per se, is what is important. This finding is linked to the multi-firm static model results by exploiting a chain of comparisons between the Markov perfect equilibrium, the commitment equilibrium, and the multi-firm static Nash-Cournot equilibrium.

Note that the specification of preferences here subsumes durable goods as a special case. That is, when $\delta = 0$ (which implies $A < 0$), the preference structure can be thought of as reflecting utility associated with a durable good for which a consumer incurs adjustment costs from changes in the size of his or her stock. Given the prevalence of durable goods in international trade and the prevalence of oligopolistic market structure for these goods, this suggests these results are more than a theoretical curiosum.

Casual observation also suggests that many products might fit in the "addictive" or "experience" goods category, and are also produced in oligopolistic settings. Thus these results might have wide applicability.

Finally, by providing an expanded array of circumstances in which the optimal export policy might call for a tax rather than a subsidy, this analysis might help explain international firm location decisions. In particular, when the optimal policy is a tax, if part or all of the tax revenue could be rebated to the firms in a lump-sum manner, the home country firm(s) would be better off than without a tax at all. Thus, if in contrast to other

countries, a country with both per-unit and lump-sum instruments at their disposal could offset more fundamental cost advantages offered by rival foreign locations, such as a lack of pollution regulations.

References

Becker, G. and K. Murphy, 1988, "A theory of rational addiction," *Journal of Political Economy*, 96 (4), (August), 675–700.

Brander, J., 1995, "Strategic Trade Policy," in G. Grossman and K. Rogoff (eds), *Handbook of International Economics*, Vol. III, Elsevier Science B.V., Amsterdam, The Netherlands, 1395–1455.

Brander, J. and B.J. Spencer, 1985, "Export subsidies and international market share rivalries," *Journal of International Economics*, XVIII, 83–100.

Coase, R., 1972, "Durability and Monopoly," *Journal of Law and Economics*, 15, 143–149.

De Santis, R.A., 2000, "Optimal Export Taxes, Welfare, Industry Concentration, and Firm Size: A General Equilibrium Analysis," *Review of International Economics*, 8(2), 319–335.

Dixit, A., 1984, "International trade policy for oligopolistic industries," *Economic Journal*, 94 (supplement), March, 1–16.

Dockner, E.J. and A.A. Haug, 1990, "Tariffs and quotas under dynamic duopolistic competition," *Journal of International Economics*, Vol. 29, 147–160.

Driskill, R. and A. Horowitz, 1996, "Durability and strategic trade: are there rents to be captured?", *Journal of International Economics*, Vol. 41, Nos. 1/2 (August 1996), 179–194.

Driskill, R. and S. McCafferty, 1989, "Dynamic duopoly with output adjustment costs in international markets: taking the conjecture out of conjectural variations," in R.C. Feenstra (ed), *Trade Policies for International Competitiveness*, University of Chicago Press, Chicago.

Driskill, R. and S. McCafferty, 2001, "Monopoly and Oligopoly Provision of Addictive Goods," *International Economic Review*, Vol. 42, No. 1, (February), 43–72.

Eaton, J. and G. Grossman, 1986, "Optimal trade and industrial policy under oligopoly," *Quarterly Journal of Economics*, 101, 383–406.

Krishna, K. and M. Thursby, 1991, "Optimal policies with strategic distortions," *Journal of International Economics*, Vol. 31, 291–308.

Leary, D. and J. Peter Neary, 1999, "Learning by Doing, Precommitment and Infant-Industry Promotion," *Review of Economic Studies*, Vol. 66 (April), 447–474.

Pollak, R., 1970, "Habit Formation and dynamic demand functions," *Journal of Political Economy*, 78 (4), 745–763.

Reinganum, J.F. and N.L. Stokey, 1985, "Oligopoly Extraction of a Common Property Natural Resource: The Importance of the Period of Commitment in Dynamic Games," *International Economic Review*, 26, 161–173.

Rodrik, D., 1989, "Optimal Trade Taxes for a Large Country with Non-Atomistic Firms," in E. Helpman and P. Krugman (eds), *Trade Policy and Market Structure*, MIT Press, Cambridge, MA.

Ryder, H.E., and G.M. Heal, 1973, "Optimum Growth with Intertemporally Dependent Preferences," *Review of Economic Studies*, 40, 1–32.

Yuen, A. and A. Zhang, 2007, "Trade, transport, and Intergovernmental Cooperation," mimeo.